

CSE 150A-250A AI: Probabilistic Models

Lecture 16

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Slides adapted from previous versions of the course (Prof. Lawrence, Prof. Alvarado, Prof Berg-Kirkpatrick)

Agenda

Review

Policy Based

Policy Evaluation

Policy Improvement

Policy Iteration

Value Iteration

Review

Value Functions

- State Value Function

$$\begin{aligned} V^\pi(s) &= \mathbb{E}^\pi \left[\sum_{t=0}^{\infty} \gamma^t R(s_t) \mid s_0 = s \right] \\ &= R(s) + \gamma \sum_{s'} P(s'|s, \pi(s)) V^\pi(s') \end{aligned}$$

- Action Value Function

$$\begin{aligned} Q^\pi(s, a) &= \mathbb{E}^\pi \left[\sum_{t=0}^{\infty} \gamma^t R(s_t) \mid s_0 = s, a_0 = a \right] \\ &= R(s) + \gamma \sum_{s'} P(s'|s, a) V^\pi(s') \end{aligned}$$

- **Goal**

Find the optimal policy given the environment that the agent is in.

- **Planning**

If reward function and transition probabilities are known.

- **Reinforcement Learning**

If reward function and transition probabilities are unknown.

Optimality



There exists **at most** one policy π^* such that $V^{\pi^*}(s) \geq V^\pi(s)$ for all policies π and states s of the MDP.

True (A) or False (B)?

Optimal value functions, $Q^*(s, a)$ and $V^*(s)$ are unique and all optimal policies share the same value functions.

True (A) or False (B)?

- Theorem

There exists at least one policy π^* (and perhaps many) such that $V^{\pi^*}(s) \geq V^\pi(s)$ for all policies π and states s of the MDP.

- Notation

$$V^*(s) = V^{\pi^*}(s)$$

$$Q^*(s, a) = Q^{\pi^*}(s, a)$$

These optimal value functions are **unique**.

(All optimal policies share the same value functions.)

We can get the optimal policy π^* from the optimal value function $V^*(s)$ but not from the optimal action value function $Q^*(s, a)$.

True (A) or False (B)?

Relations at optimality

- From the optimal action value function:

$$V^*(s) = \max_a [Q^*(s, a)]$$

$$\pi^*(s) = \operatorname{argmax}_a [Q^*(s, a)]$$

- From the optimal state value function:

$$Q^*(s, a) = R(s) + \gamma \sum_{s'} P(s'|s, a) V^*(s')$$

$$\pi^*(s) = \operatorname{argmax}_a [R(s) + \gamma \sum_{s'} P(s'|s, a) V^*(s')]$$

- Why are these relations useful?

Sometimes it can be easier to estimate $Q^*(s, a)$ or $V^*(s)$ (which are **continuous**) than to learn $\pi^*(s)$ (which is **discrete**).

Planning in MDPs

Given a complete model of the agent and its environment as a Markov decision process, namely

$$\text{MDP} = \{\mathcal{S}, \mathcal{A}, P(s'|s, a), R(s), \gamma\},$$

how can we *efficiently* compute (i.e., in time *polynomial in the number of states*) any of the following:

1. an optimal policy $\pi^*(s)$?
2. the optimal state value function $V^*(s)$?
3. the optimal action value function $Q^*(s, a)$?

This is the problem of **planning** in MDPs.

Policy Based

1. Policy evaluation

How to compute $V^\pi(s)$ for some fixed policy π ?

2. Policy improvement

How to compute a policy π' such that $V^{\pi'}(s) \geq V^\pi(s)$?

3. Policy iteration

How to compute an optimal policy $\pi^*(s)$?

Policy evaluation

- How to compute the state value function?

$$V^\pi(s) = \mathbb{E}^\pi \left[\sum_{t=0}^{\infty} \gamma^t R(s_t) \mid s_0 = s \right]$$

$R(s, s', a)$

- Bellman equation:

$$V^\pi(s) = R(s) + \gamma \sum_{s'} P(s'|s, \pi(s)) V^\pi(s')$$

- Solve linear system: There are n equations for n unknowns (where $s = 1, 2, \dots, n$).

Solving the linear system (con't)

- Solution

$$R = [I - \gamma P^\pi] V^\pi \implies V^\pi = \underbrace{(I - \gamma P^\pi)^{-1}}_{\text{matrix inverse}} R$$

- Complexity

It takes $O(n^3)$ operations to solve this system of equations.

- Problem statement

Given a policy π and its state value function $V^\pi(s)$,
how to compute a policy π' such that

$$V^{\pi'}(s) \geq V^\pi(s) \quad \text{for all states } s?$$

- Definition

Given the action value function $Q^\pi(s, a)$ for policy π , we
define the **greedy policy** π' by

$$\pi'(s) = \operatorname{argmax}_a \left[Q^\pi(s, a) \right].$$

action ✓ -

- In terms of the state value function:

$$\begin{aligned}\pi'(s) &= \operatorname{argmax}_a \left[Q^\pi(s, a) \right] \\ &= \operatorname{argmax}_a \left[R(s) + \gamma \sum_{s'} P(s'|s, a) V^\pi(s') \right] \\ &= \operatorname{argmax}_a \left[\sum_{s'} P(s'|s, a) V^\pi(s') \right]\end{aligned}$$

- Test your understanding:

$\pi'(s) = \pi(s)$ for some $s \in \mathcal{S}$? not necessarily

$\pi'(s) \neq \pi(s)$ for some $s \in \mathcal{S}$? not necessarily

$Q^\pi(s, \pi'(s)) \geq Q^\pi(s, \pi(s))$ for all $s \in \mathcal{S}$? TRUE

Policy improvement

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If it's better to choose action a in state s before following π , then it's always better to make this choice.

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- Intuition:

If it's better to choose action a in state s before following π , then it's always better to make this choice.

- Proof idea:

We'll prove a key inequality for *one-step deviations* from π , then we'll extend this inequality by an iterative argument.

Proof – 1. Deriving the inequality

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- Comparing value functions:

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$$V^\pi(s) = Q^\pi(s, \pi(s))$$

action that comes from π

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$\pi' \leftarrow \pi$

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$$V^\pi(s) \leq R(s) + \gamma \sum_{s'} P(s'|s, \pi'(s)) V^\pi(s')$$

- Intuition:

It is better to take one step under π' , then revert to π , than to always follow π .

Proof – 2. Leveraging the inequality

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$$V^\pi(s) \leq R(s) + \gamma \sum_{s'} P(s'|s, \pi'(s)) \left[R(s') + \gamma \sum_{s''} P(s''|s', \pi'(s')) V^\pi(s'') \right]$$

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- Intuition:

It is better to take **two** steps under π' , then revert to π , than to always follow π .

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It is better to take t steps under π' , then revert to π , than to always follow π . Last term is of order $O(\gamma^t)$.

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- Take the limit $t \rightarrow \infty$:

It is better to follow π' (always) than to follow π (always).

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- Apply the inequality t times:

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- Take the limit $t \rightarrow \infty$:

It is better to follow π' (always) than to follow π (always). Conclude that $V^\pi(s) \leq V^{\pi'}(s)$ for all states $s \in \mathcal{S}$.

Policy iteration

How to compute π^* ?

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$$\pi_0 \xrightarrow{\text{evaluate}} \begin{matrix} V^{\pi_0}(s) \\ Q^{\pi_0}(s, a) \end{matrix}$$

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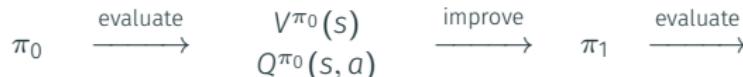
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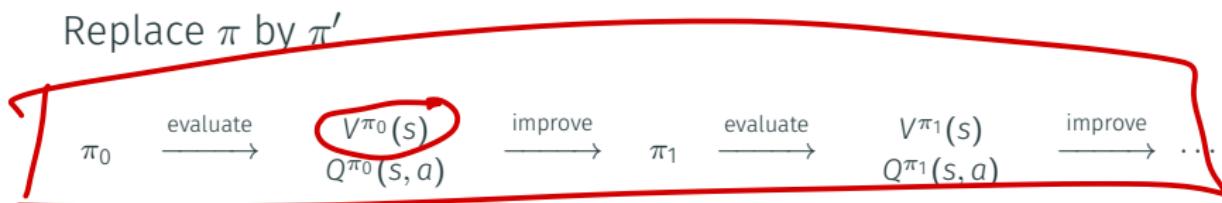
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Policy iteration is guaranteed to terminate.

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Prove a key **equality/inequality** for **terminal/non-terminal** policies;

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- Proof idea

Prove a key **equality/inequality** for **terminal/non-terminal** policies;
iterate t times, then compare the limits as $t \rightarrow \infty$.

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$$V^{\pi}(s) = R(s) + \gamma \max_a \sum_{s'} P(s'|s, a) V^{\pi}(s')$$

These equations are **nonlinear** due to the **max** operation.

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at convergence

Now exploit that π' is greedy with respect to π ...

- Bellman optimality equation

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These equations are **nonlinear** due to the **max** operation.

There are n equations for n unknowns (where $s = 1, 2, \dots, n$).

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The BOE only holds for a solution π from policy iteration.

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Since $\tilde{\pi}$ is arbitrary, we conclude that π is optimal.

Value Iteration

Motivation

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So if we can directly compute the optimal value function $V^*(s)$, then we can use it to derive an optimal policy π^* .

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But how to solve these **nonlinear** equations for $V^*(s)$?

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Annotations:

- Red double arrows:** Point to the $V_{\text{new}}(s)$ term on the left.
- Orange arrow:** Points to the \leftarrow symbol.
- Red bracket:** Brackets the entire right-hand side of the equation, from $R(s)$ to $V_{\text{old}}(s')$.
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The value function $V^*(s)$ is a *fixed point* of this iteration.
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PI converges in a finite number of steps.

VI converges asymptotically (in the limit).

That's all folks!